

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT3

Branch: Diploma (All)

Semester : 2

Date : 20/04/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{5\theta} = \underline{\hspace{2cm}}$
(A) 1 (B) 0 (C) 1/5 (D) 5
- b) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\frac{x}{2}} = \underline{\hspace{2cm}}$
(A) e^2 (B) e (C) $e^{1/2}$ (D) None of these
- c) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \underline{\hspace{2cm}}$
(A) -1 (B) 1 (C) 0 (D) 3
- d) $\frac{d(7)}{dx} = \underline{\hspace{2cm}}$
(A) -1 (B) 0 (C) 1 (D) None of these
- e) $\frac{d(1/x)}{dx} = \underline{\hspace{2cm}}$
(A) $-\frac{1}{x^2}$ (B) $\frac{1}{x}$ (C) e (D) 1
- f) $\frac{d(e^{5x})}{dx} = \underline{\hspace{2cm}}$
(A) $5e^x$ (B) e^x (C) $\frac{e^{5x}}{5}$ (D) $5e^{5x}$
- g) $\frac{d(\cot^{-1} x)}{dx} = \underline{\hspace{2cm}}$
(A) $\frac{-1}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$



- h) $\int 1 dx = \underline{\hspace{2cm}}$
 (A) 0 (B) 1 (C) $x+c$ (D) None of these
- i) $\int a^x dx = \underline{\hspace{2cm}}$
 (A) $\frac{a^x}{\log_e a} + c$ (B) $a^x \log_e a + c$ (C) $a^x + c$ (D) $\log_e a + c$
- j) $\int_0^1 \frac{4}{1+x^2} dx = \underline{\hspace{2cm}}$
 (A) π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) None of these
- k) $|(2, 1, 1) + (1, 2, 3)| = \underline{\hspace{2cm}}$
 (A) 10 (B) $\sqrt{43}$ (C) $\sqrt{34}$ (D) None of these
- l) If $\bar{a} = i - j + k$ then $\hat{a} = \underline{\hspace{2cm}}$
 (A) $\frac{(1, -1, 1)}{\sqrt{3}}$ (B) $\frac{(-1, 1, 1)}{\sqrt{3}}$ (C) $\frac{(1, 1, -1)}{\sqrt{3}}$ (D) $\sqrt{3}$
- m) If $\bar{x} = (1, 1, 1)$ and $\bar{y} = (2, -1, 3)$ then $\bar{x} \times \bar{y} = \underline{\hspace{2cm}}$
 (A) $(4, -1, 3)$ (B) $(-4, -1, 3)$ (C) $(-4, 1, 3)$ (D) $(4, -1, -3)$
- n) If $\bar{x} = (1, 1, 1)$ and $\bar{x} = (2, -2, 1)$ then $\bar{x} \cdot \bar{y} = \underline{\hspace{2cm}}$
 (A) 5 (B) 1 (C) 0 (D) -1

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) If $\bar{a} = 3i - 2j + k$, $\bar{b} = 2i - 4j - 3k$ and $\bar{c} = -i + 2j + 2k$ (5)
 then find modulus of $2\bar{a} - 3\bar{b} - 5\bar{c}$.
- b) Find unit vector which is perpendicular to both $x = (3, 1, 2)$ and (5)
 $y = (2, 1, 1)$.
- c) Evaluate: $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$ (4)

Q-3 Attempt all questions (14)

- a) Prove that $\lim_{x \rightarrow a} \frac{\sqrt{2a-x} - \sqrt{x}}{a-x} = \frac{1}{\sqrt{a}}$. (5)
- b) Evaluate: $\lim_{x \rightarrow 0} \frac{x \tan x}{1 - \cos x}$ (5)
- c) Find $\frac{dy}{dx}$ if $y = \frac{a + b \sin x}{a \sin x + b}$. (4)

Q-4 Attempt all questions (14)

- a) $S = t^3 - 6t^2 + 8t - 4$ gives the distance travelled by a body in t seconds. Find (5)
 velocity and acceleration at $t = 4$ second.



b) Find $\frac{dy}{dx}$ if $y = \log\left(\frac{\sin x}{1 + \cos x}\right)$. (5)

c) If $\bar{a} = 2i - j$, $\bar{b} = i + 3j - 2k$ then obtain $\left|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})\right|$. (4)

Q-5

Attempt all questions

(14)

a) Prove that angle between two vectors $3i + j + 2k$ and $2i - 2j + 4k$ is (5)

$$\sin^{-1}\left(\frac{2}{\sqrt{7}}\right).$$

b) If $y = 2e^{-3x} + 3e^{2x}$ then prove that $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$. (5)

c) Evaluate: $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + \dots + n^3}{n^2(1 + 2 + \dots + n)}$ (4)

Q-6

Attempt all questions

(14)

a) A particle moves from the point $-i - j - k$ to the point $k + j + i$ under the effect of two constant forces $2i + j + k$ and $i + 3j + k$. Find the work done. (5)

b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$. (5)

c) Find derivative of $y = x^3 - 2x$ using first principle. (4)

Q-7

Attempt all questions

(14)

a) Evaluate: $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx$ (5)

b) Find the volume of sphere of radius r . (5)

c) For what value of m , vectors $mj + 2i + k$ and $2i + 4j + 5k$ are perpendicular to each other? (4)

Q-8

Attempt all questions

(14)

a) Evaluate: $\int x^n \log x dx$ (5)

b) Find $\frac{dy}{dx}$ if $y = (e^{3x} + 1)^{2x+5}$. (5)

c) Evaluate: $\int_0^{\frac{\pi}{3}} \frac{\sin x}{3 + 4 \cos x} dx$ (4)

